ANALYSIS OF SWEPT TIME AND MODAL PARAMETER EFFECT ON FRF’S MAGNITUDE ERROR OF SDOF SYSTEM USING LINEAR SWEPT-SINE EXCITATION

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Abstract
Random and swept-sine excitations are the most commonly used excitations in FRF (Frequency Response Function) measurement. There are various type of swept-sine excitation can be used, however, linear swept-sine is often used in FRF measurement. Linear swept-sine excitation is a sinusoidal excitation of which its frequencies change linearly within time. This paper presents swept time and modal parameter effect on FRF’s magnitude error of SDOF (Single Degree of Freedom) system using linear swept-sine excitation. FRF’s magnitude error is focused at the peak of FRF’s magnitude error (system’s resonant frequency). Based on analysis study, the peak of FRF’s magnitude error of SDOF system is a function of swept time and system’s modal parameter (resonant frequency and damping ratio).

Keywords: FRF, linear swept-sine excitation, SDOF system, the peak of FRF’s magnitude error.

I. INTRODUCTION
FRF (Frequency Response Function) measurement is conducted to depict dynamic characteristic of a system. In the FRF measurement, the system is vibrated by using exciter. Both excitation force and system’s vibration response are measured concurrently. By using signal’s frequencies analyzer, both measured signals are transformed into force’s frequencies spectrum and response frequencies spectrum respectively. Ratio of response frequencies spectrum magnitude to force’s frequencies spectrum magnitude is known as FRF’s magnitude of the system (McConnell, 1995). The most commonly used exciter in the FRF measurement is impact hammer or shaker. For shaker excitation, some equipment’s are needed. They are signal generator, power amplifier, shaker, and stinger (Peres et.al, 2010; Cloutier et.al, 2009; Füllekrug et.al., 2008). The advantage of using shaker’s excitation is its frequencies can be controlled very well.

There are various type of shaker’s excitations can be used, however, swept-sine excitation is the most common type of excitation in the FRF measurement (Zhuge, 2009; Peeters et.al., 2008; Climent, 2007; Göge et.al., 2007; Pauwels et.al., 2006; Schwarz, 1999). Swept-sine excitation is a sinusoidal excitation of
which its frequencies change within time or depend on swept function. If swept-sine excitation has linear swept function, then, it is called with linear swept-sine. It is a kind of swept-sine excitation that often be used in FRF measurement (Orlando et.al., 2008; Gloth et.al., 2004(a), 2004(b); Baoliang et.al., 2003; Haritos, 2002).

FRF’s magnitude of a system at certain frequency span can be obtained from FRF measurement using linear swept-sine excitation for different swept time. The swept time is duration that needed for exciting the system in FRF measurement at certain frequency span. For tested system as well as SDOF (Single Degree of Freedom) system, it can be classified based on its modal parameter (resonant frequency and damping ratio). To better understand swept time and modal parameter effect on FRF’s magnitude error of SDOF system, an analysis study is presented in this paper. Here, frequency span value is limited only 40 Hz. This value is chosen because the first resonant frequency of the system often lies under it. FRF’s magnitude error of the system is focused at the peak of FRF’s magnitude error (system’s resonant frequency).

II. METHODOLOGY

Analysis study to understand swept time and modal parameter effect on FRF’s magnitude error of SDOF system using linear swept-sine excitation is conducted through a numerical simulation. The simulation contains some stages in flowchart as shown in Figure 1.

![Flowchart of analysis study to understand swept time and modal parameter effect on FRF’s magnitude error of SDOF system using linear swept-sine excitation.](image-url)

Figure 1. Flowchart of analysis study to understand swept time and modal parameter effect on FRF’s magnitude error of SDOF system using linear swept-sine excitation.
System mass $m$, frequency span $f_r$, amplitude of excitation force $A$, resonant frequency $f_r$, damping ratio $\zeta$, and swept time $T_s$ are simulation’s input parameters. For each of various resonant frequencies $f_r$, system stiffness $k$ and system damping $c$ respectively are

$$k = \frac{4\pi^2 mgf^2}{(1-\zeta^2)}$$  \hspace{1cm} (1)

$$c = 2\zeta \sqrt{mk}$$  \hspace{1cm} (2)

Then, the theoretical FRF’s magnitude of SDOF system $|H_{teo}(f)|$ is obtained.

The linear swept-sine excitation $u(t)$ has starting frequency $f_0$, ending frequency that equal to span frequency $f_r$, swept-time $T_s$, and amplitude $A$ (Yanto et. al., 2012a). The $u(t)$ is expressed with

$$u(t) = A \sin \left( \pi \frac{(f_0 - f_r) t^2}{2T_s} + 2\pi f_0 t \right)$$  \hspace{1cm} (4)

If a SDOF system is excited by using $u(t)$, then, it is modeled as the impulse response function $h(t)$. Figure 2 shows the $h(t)$ input-output relationship (Yanto et. al., 2012b).

![Diagram of SDOF system](image)

**Figure 2.** The $h(t)$ input-output relationship.

The system state $x(t)$ and output $y(t)$ can be written as follow:

$$\begin{bmatrix} x_2(t+1) \\ x_2(t+1) \end{bmatrix} = e^{-a\Delta t - b\Delta t} \begin{bmatrix} \frac{k}{m} & \frac{1}{m} \\ -\frac{k}{m} & -\frac{a}{m} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \frac{1}{m(b-a)} \left\{ \frac{1}{a}(e^{-\alpha \Delta t} - 1) + \frac{1}{a}(e^{-\beta \Delta t} - 1) \right\} u(t)

$$  \hspace{1cm} (5)

$$y(t) = x_2(t) \hspace{1cm} 0 \leq t_i \leq T_r$$  \hspace{1cm} (6)

Where:

$$\alpha = \zeta \sqrt{\frac{k}{m}} + j \sqrt{\frac{c}{m}} (1 - \zeta^2)$$

$$\beta = \zeta \sqrt{\frac{k}{m}} - j \sqrt{\frac{c}{m}} (1 - \zeta^2)$$

$$\Delta t = \frac{1}{4\phi f_0}$$  \hspace{1cm} : Sampling period

$$\begin{bmatrix} x_1(t_0) \\ x_2(t_0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$  \hspace{1cm} : Initial conditions

Equations (5) and (6) are the equations for simulating SDOF system vibration using linear swept-sine excitation. The simulation of SDOF system vibration is conducted to obtain both numerically system’s vibration response $y(t)$ signal. Then, both $u(t)$ and $y(t)$ signals are transformed into force’s
frequencies spectrum $U(f)$ and response frequencies spectrum $Y(f)$ by using Fourier Transform Method respectively as shown in Figure 3. The FRF is obtained by

$$H(f) = \frac{G_y(f)}{G_u(f)}$$

(7)

Where:

$G_y(f) = U^*(f)Y(f)$ : The Cross Power Spectrum between $u(t)$ and $y(t)$.

$G_u(f) = U^*(f)U(f)$ : The Auto Power Spectrum of $u(t)$.

Figure 3. The dual channel frequency analysis.

The FRF’s magnitude error is focused at system’s resonant frequency that be determined by

$$E_{m}(f_r) = \frac{\left|H_{teo}(f_r)\right| - \left|H(f_r)\right|}{\left|H_{teo}(f_r)\right|} \cdot 100\%$$

(8)

III. RESULTS AND DISCUSSION

Value of input parameters in analysis study of swept time and modal parameter effect on FRF’s magnitude error of SDOF system are shown in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>1 kg</td>
<td></td>
</tr>
<tr>
<td>$f_c$</td>
<td>40 Hz</td>
<td></td>
</tr>
<tr>
<td>$A$</td>
<td>1 N</td>
<td></td>
</tr>
<tr>
<td>$f_r$</td>
<td>4 To 36 Step 4 Hz</td>
<td></td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.5 To 1 Step 0.1 %</td>
<td></td>
</tr>
<tr>
<td>$T_r$</td>
<td>1 To 30 Step 1 s</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Value of input parameters.

Figure 4 shows an example of the theoretical FRF’s magnitude of SDOF system where $m = 1$ kg, $f_c = 16$ Hz, $\zeta = 1\%$, and $f_e = 40$ Hz are known.

If linear swept-sine excitation has $f_0 = 0$ Hz, $f_e = 40$ Hz, and $T_r = 4$ s, so, it can be described as shown in Figure 5.

The linear swept-sine excitation in Figure 5 is applied on the SDOF system of which $m = 1$ kg, $f_c = 16$ Hz, and $\zeta = 1\%$ would cause system’s vibration response as shown in Figure 6.

Figure 4. The theoretical FRF’s magnitude of SDOF system where $m = 1$ kg, $f_c = 16$ Hz, $\zeta = 1\%$, and $f_e = 40$ Hz are known.

Figure 5. Linear swept-sine excitation where $f_0 = 0$ Hz, $f_e = 40$ Hz, and $T_r = 4$.

Figure 6. Vibration response of SDOF system ($m = 1$ kg, $f_c = 16$ Hz, and $\zeta = 1\%$) on linear swept-sine excitation ($f_0 = 0$ Hz, $f_e = 40$ Hz, and $T_r = 4$).
The FRF’s magnitude of of SDOF system \((m = 1 \text{ kg}, f_r = 16 \text{ Hz}, \text{ and } \zeta = 1\%)\) using linear swept-sine excitation \((f_0 = 0 \text{ Hz}, f_e = 40 \text{ Hz}, \text{ and } T_r = 4)\) is compared with its theoretical FRF’s magnitude can be illustrated in Figure 7.

For SDOF system of which value of its resonant frequency \(f_r\) and damping ratio \(\zeta\) are varied as in Table 1, the peak of FRF’s magnitude error \(E_{\alpha}(f_r)\) are shown in Figure 8. This error is a function of swept time and system’s modal parameter (resonant frequency and damping ratio) that can be expressed by

\[
E_{\alpha}(f_r) = f(T_r, f_r, \zeta)
\]  

Values of \(E_{\alpha}(f_r)\) are more influenced by swept time \(T_r\) for each of system’s modal parameter. If linear swept-sine excitation is applied to SDOF system with swept time \(T_r\) equal to or large than 23 s, then, obtained \(E_{\alpha}(f_r)\) less than 10 % as shown in Figure 9.

IV. CONCLUSION

FRF’s magnitude error of SDOF systems using linear swept-sine excitation are effected by swept time and system’s modal parameter. FRF’s magnitude error less than 10 % can be obtained by using linear swept-sine excitation that has swept time equal to or large than 23 seconds for each of system’s modal parameter.

Figure 7. The FRF’s magnitude of SDOF system using linear swept-sine excitation is compared with its theoretical FRF’s magnitude.

Figure 8. The FRF’s magnitude error SDOF system using linear swept-sine excitation to its theoretical FRF’s magnitude.
Figure 9. Swept time $T_r$ effect on values of $E_a(f_r)$ for each system’s modal parameter.

This analysis study can be used as a reference in FRF measurement using linear swept-sine excitation to avoid large error of FRF’s magnitude of the testing system experimentally.

REFERENCES


the ISMA 2008 International Conference on Noise and Vibration Engineering, Leuven.


